ISSN 2348 – 8034
Impact Factor- 5.070

GLOBAL JOURNAL OF ENGINEERING SCIENCE AND RESEARCHES

RANDOM NUMBERS VS STOCHASTIC MATRIX IN MARKOV CHAIN FOLLOWED BY STRENGTHS AND WEAKNESS IN INNOVATIONAL APPROACH

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ABSTRACT
A Stochastic process is defined to be an indexed collection of random variables {Xt},where the index t runs through a given set T.Often T is taken to be the set of non-negative integers and Xt represents a measurable characteristic of interval of time t. For example Xt might represent the inventory level of a particular product at the end of the week t.In this paper we have used the concept of present and future to formulate Stochastic Matrix with the help of random words and numbers which will be helpful to undergraduate engineering students to understand the concept of Stochastic Matrix through the concept provided in "Random Words Vs Stochastic Matrix in Markov Chain followed by strengths and weakness in innovational approach'' in Markov Chain as a Learning Tool.

Keywords: *Stochastic Process, random variables, Markovian Property, Stochastic Matrix and Inventory Model*

I. INTRODUCTION

A stochastic process is defined to be an indexed collection of random variables where the index t runs through a given set T. T is considered to be the set of non-negative integers, and X_t represents a measurable characteristic of interest at time t. For example X_t might represent the inventory level of a particular product at the end of the week t. A stochastic process $\{X_t\}$, t = 0, 1, 2 … is a Markov Chain if it has the Markovian Property.

Markovian Property : X_{t+1}, the state of the system at time t+1 depends on the state of the system at time t i.e. $P\{X_{t+1} = j|X_0 = k_0, X_1 = k_1, X_2 = k_2 \ldots X_{t-1} = k_{t-1}, X_t = i\} = P\{X_{t+1} = j|X_t = i\}$ for t = 0, 1, …, and for every sequence i, j, $k_0, k_1, \ldots k_{t-1}.$ Markovian Property says that the conditional probability of any future event given any past event and the present state $X_t = i$ is independent of past event and depends upon the present state [2].

II. TRANSITION PROBABILITIES (MATHEMATICAL DESCRIPTIONS)

The conditional probabilities $P\{X_{t+1} = j|X_t = i\}$ for a Markov Chain are called transition probabilities. If for each i and j $P\{X_{t+1} = j| X_t = i\} = P\{X_1 = j| X_0 = i\}$ for all t=1,2,…,then 1 – step transition probabilities are said to be stationary.

Markov Process:
Simple Example: (Weather):

Raining today ⇒ 30 % rain tomorrow
⇒ 70 % no rain tomorrow
Not raining today ⇒ 10 % rain tomorrow
⇒ 90 % no rain tomorrow

We can represent this by stochastic finite state.

$$\begin{array}{c c} & \begin{array}{cc} \text{Rain} & \text{No rain} \end{array} \\ \begin{array}{c} \text{Rain} \\ \text{No Rain} \end{array} & \begin{bmatrix} 0.3 & 0.7 \\ 0.1 & 0.9 \end{bmatrix} \end{array}$$

n-step transition probabilities:
It is represented by $P^n = P^{n-1}P$ or $P^n = P^{n-m}P^m$, $0<m<n$.

Application 1. Every year at the beginning of the season (through March to September) gardener uses chemical test to check soil condition. Depending on the outcome of the test productivity for the next season falls on in one of the three states (1) good (2) fair (3) poor. Definitely it is found that last year soil condition impacts current year's productivity. The situation can be described by the following Markov Chain [4].

$$P = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{pmatrix} 0.2 & 0.5 & 0.3 \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 1 \end{pmatrix} \end{array}$$

After using fertilizer to boost the soil condition, the modified transition matrix becomes

$P_1=\begin{pmatrix} .30 & .60 & .10 \\ .10 & .60 & .30 \\ .05 & .40 & .55 \end{pmatrix}$.Now it is very clear that there is an improvement in the deteriorating condition. Using the concept of transition probabilities we can calculate states of the system after1, 8 and 16 gardening seasons i.e. $a^{(1)}$ for the 1st gardening season is

$a^{(1)} = (1 \quad 0 \quad 0)P_1 = (.30 \quad .60 \quad .01)$.

Similarly

$a^{(8)} = (1 \quad 0 \quad 0)P^8 = (.101753 \quad .522514 \quad .372733)$.

Similarly $a^{16} = (1 \quad 0 \quad 0)p^{16} = (.101659 \quad .525450 \quad .372881)$.

Application 2. Jones Charles in 1997 investigated the relative movements of countries between income brackets. He used a Markov Chain technique based in the preceding discussion to show how nations moved between different income categories in the period from 1960 to 1988.It was modified by Michael Zabek by using proportions of countries in each category and proportions of people in each category specified by high. medium and low. Both of them calculated real GDP to that of the US in each period. He was able to do this by using a regular Markov Chain i.e. no zero entries in the corresponding matrix [1].

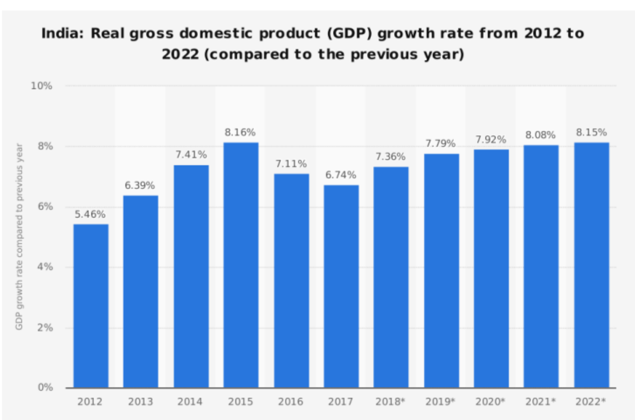

Fig.1

III. STOCHASTIC MATRIX

In Mathematics Stochastic Matrix is a square a matrix used to describe the transitions of a Markov Chain. Each of the entries is a non-negative real number representing a probability. It is also called as Probability Matrix or Substitution Matrix or Markov Matrix.

IV. HOW TO OBTAIN STOCHASTIC MATRIX (HISTORY)

We can learn from examples:
Which letter follow which letters in English words mast, tame, same, teams, team, meat, steam and stem?

Table-I

	a	s	t	M	e	Ends with	Sum
a	-	1	1	5	-	-	7
s	1	-	3	-	-	1	5
t	1	-	-	-	4	2	7
m	1	1	-	-	3	3	8
e	4	-	-	1	-	2	7
Start with	-	3	3	2	-	-	8

The above matrix represents the concept of Markovian property by following **"first then next".**

Stochastic Matrix
Table-II

P	a	s	t	m	e	Ends with
a	0	1/7	1/7	5/7	0	0
s	1/5	0	3/5	0	0	1/5
t	1/7	0	0	0	4/7	2/7
m	1/8	1/8	0	0	3/8	3/8
e	4/7	0	0	1/7	0	2/7
Start with	0	3/8	3/8	2/8	0	0

This paper was published in International Journal of Scientific and Innovative Research (IJSR) Sky Institute of India Vol 2.May 1,2016(by Isabel Griffith) [5].

V. ANALYSIS BY AUTHORS

We have used the same concept of Markovian Property to formulate Stochastic Matrix.

Analysis: We used eight numbers by using two single digits 1 and 5 and three double digits 13.19 and 20.The respective numbers are: (i)13 1 19 20(ii)20 1 13 5 (iii)19 1 13 5 (iv)20 5 1 13 19 (v)20 5 1 13 (vi)13 5 5 20 (vii)19 20 5 1 13 (8)19 20 5 13.

ISSN 2348 – 8034
Impact Factor- 5.070

Formulation of Matrix
Table-III

		NEXT						
		1	5	13	19	20	Ends with	sum
FIRST	1	0	0	5	1	0	0	6
	5	1	0	1	0	1	2	5
	13	1	3	0	1	0	3	8
	19	1	0	0	0	1	1	3
	20	0	4	0	0	0	2	6
	Start with	0	0	2	3	3	0	8

Stochastic Matrix
Table-IV

		NEXT					
		1	5	13	19	20	Ends with
FIRST	1	0	0	5/6	1/6	0	0
	5	1/5	0	1/5	0	1/5	2/5
	13	1/8	3/8	0	1/8	0	3/8
	19	1/3	0	0	0	1/3	1/3
	20	0	4/6	0	0	0	2/6
	Start with	0	0	2/8	3/8	3/8	0

Rules to generate a random number using Stochastic Matrix:

(i) Generate most likely first digit i.e. 1.
(ii) For each current digit generate most likely next digit.
(iii) Generate a **random** number X between 1 to 20.If $1<X<13$, the single digit what we have to select is 5.if $5<X<19$, the two digit number what we have to select is 13. If $5<X<19$, the two digit number what we have to select is 13.Similarly $13<X<20$, the corresponding number is 19.Finally if $X = 20$, that is the last two digit number.

VI. WHAT ARE THE NEXT POSSIBILITIES?

(i) Following statistical table of Random Numbers we can also select sequence of random numbers 47556 75123 41456 76358 95294 consisting of the digits 1, 2,3,4,5,6,7,8 and 9 to formulate Stochastic Matrix.
(ii) We will be able to prepare a paper by using an important technique in Linear Algebra-i.e the use of Transition Matrices and Markov Chain – to address a longstanding question "especially our challenge of tracking nation's movements between different levels of income per capita.

VII. LEARNING

(i) Each and every concept can be reflected in a right way through live examples.
(ii) Students will be able to learn regarding scientific and innovative research.
(iii) Teaching must be creative instead of robotic.
(iv) Vital statistics: Source-Thomson Reuters report on innovation performance, March 2014 is as follows:

ISSN 2348 – 8034
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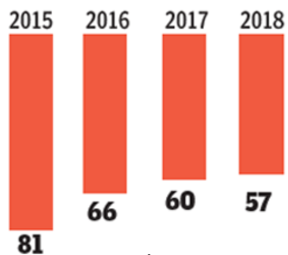
Name of the country	Percentage
EU (European Union)	40
China	25
US (United States)	22
India	13

Recent Data on Global Innovation Index (GII):
In recently released Global Innovation Index (GII) 2018.India was ranked 57th among 130 countries. It was 11th edition of GII and was jointly released by Cornell University INSEAD and World International Property Organization (WIPO).This year India has moved t up 3 places as compared to 60th rank in GII 2017 and emerged on top rank economy in central Asia and South Africa [3].

India climbs up GII rankings



Eig.2

Top 10 countries (Rank in 2018):
They are (1) Switzerland, (2) Netherlands, (3) Sweden, (4) UK, (5) Singapore, (6) USA, (7) Finland, (8) Denmark, (9) Germany, (10) Ireland.

Rank of BRICS nations in 2018

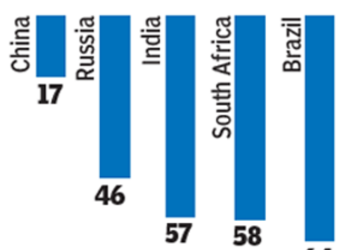


Fig.3

VIII. CONCLUSIONS

Strengths: These indicators have helped India to improve its ranking. It includes India`s human capital (graduates in Science and Engineering) growth rate of GDP per worker, experts of information and communication techniques (ICT) and services, productivity growth and creative goods expert etc.

Weakness: India has fared badly on indicators such as case of stationary business, political stability and safety, overall education and environment. So we all must join our hands in innovational research activities to overcome weakness and to see our country in much better position.

ISSN 2348 – 8034
Impact Factor- 5.070

IX. ACKNOWLEDGEMENT

We would like to thank our colleagues for their useful comments and invaluable help in the revision of the text

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